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TWO VARIABLE MODELLING OF EQUATORWARD
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This report summarizes a procedure for two variable fitting of the equatorward boundary of electron precipitation. A least squares minimization approach is used.

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TWO VARIABLE MODELLING OF EQUATORWARD BOUNDARY OF AURORAL PRECIPITATION

1. Introduction

The SSJ/3 detector consists of a set of two curved plate electrostatic snalyzers designed to measure the flux of precipitating electrons in the energy range from 50 eV to 20 KeV. The detectors are flown routinely on the satellites of the Defense Meteorlogical Satellite Program (DMSP). The data returned from the detectors have been used to conduct an extensive study of the systematics of the shape and motion of the equatorward boundary of auroral precipitation. The equatorward extent of auroral precipitation is a key boundary condition on the state of the earth's magnetosphere.

Each pass of the satellite over the auroral oval provides two boundary determinations. One technique for studying the boundary is to tag each pair of boundary determination with the magnetic local times in which they occur and to consider each pair of two crossings to have been made simultaneously. These pairs of points can then be separated into classes according to the magnetic local time zones associated with each boundary location. Since the orbit of the satellite is nearly sun synchronous the same pairs of magnetic local time zones are passed through each day such that the number of local time zone classes is limited.

The boundary of electron precipitation moves equatorward and poleward in response to variations in geomagnetic activity, changes in the Interplanetary Magnetic Field and many other variables. The functional form of the relationship is at best poorly known. It is of value therefore to see if the boundary locations in pairs of magnetic local times vary together in any consistent manner. One wishes to know how the geometry and size of the oval behave. By plotting the pairs of boundaries for a pair of magnetic local times and analyzing for a trend in the data this question can be addressed.

Since the boundaries should not be functionally dependent one on the other but are both rather functions of unmeasured quantities the normal linear regression techniques that assume one quantity to be dependent and one quantity to be independent are not appropriate. Also, since we are attempting to define the geometry and its variation we wish to deal with a single functional relationship in each pair of magnetic local times. For these reasons one requires a technique that simultaneously minimizes the error in both x and y quantities.

2. Mathematical Approach

Given a set of data points (xi, yi), i = 1, ... n, a least squares linear interpolation with respect to two independent variables, x and y, is desired. This is mathematically equivalent to saying the summations of the distance squared between each data coordinate and the generated fit will be minimal.

The distance between a point and a line is defined by eq. 2.1,

$$2.1 \qquad \delta = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

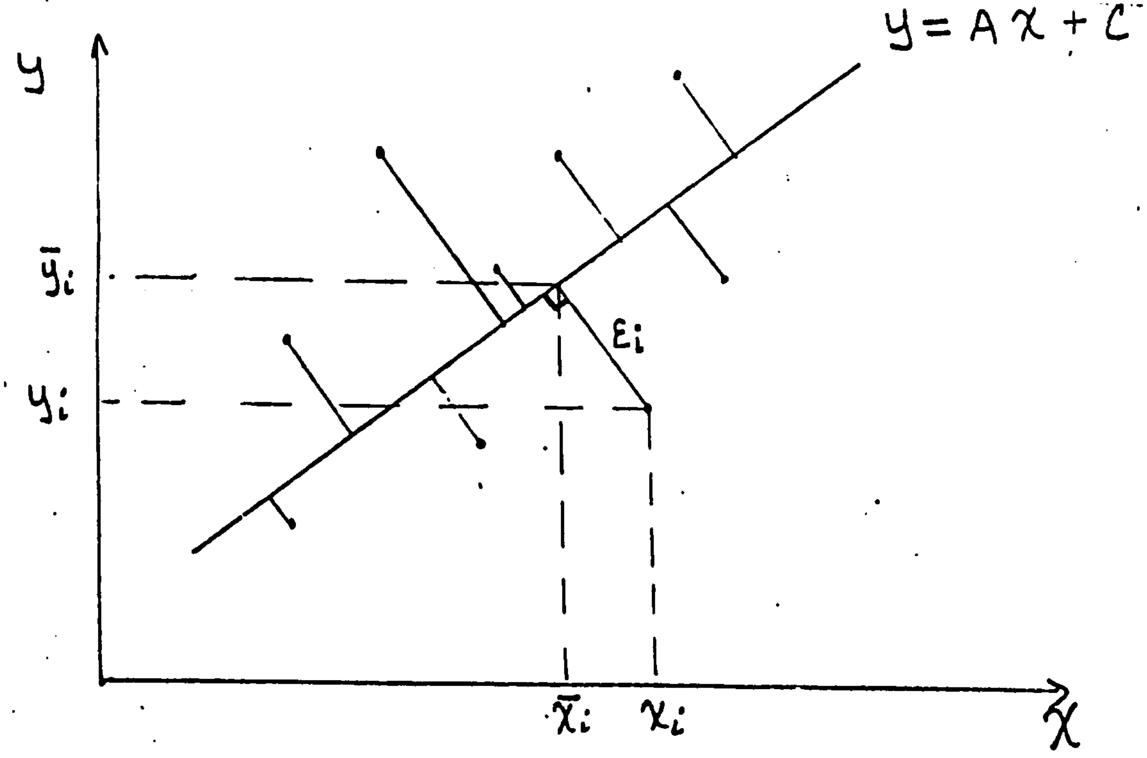
where the line is AX + BY + C = 0 and the point is (x,y)

We can now define the distance between each data point and the desired fit by eg. 2.2. Note Figure 2.1.

$$2.2 \qquad \varepsilon i = \frac{|Axi + Byi + C|}{\sqrt{A^2 + B^2}} \quad , i = 1, \ldots, n.$$

The fit to be determined will be in the conventional linear equation Y = A X + C. If the solution is going to be in this form, then "B" in eq. 2.2 takes on the value -1. This yields eq. 2.3.

Fig. 2.1 The data points do no have to be equally spaced with reference to x or y.



By a Least Squares approach determine A & C

Such that $\sum_{i=1}^{n} \varepsilon i^2$ is a minimum

$$2.3 \qquad \varepsilon i = \frac{|A\,xi - yi + C|}{\sqrt{A^2 + 1}}, \quad i = 1, \ldots, n.$$

It should be noted that we can solve for the equation X = B Y + C, yielding the distance equation

$$2.3b \qquad \varepsilon i = \frac{|-xi + B\,yi + C|}{\sqrt{1 + B^2}}, \quad i = 1 \ldots, n.$$

Either method yields the same linear fit, therefore convention shall be the rule.

Applying the "standard lease squares method" we shall minimize the squared distance between the fit and data. Thus we want to minimize eq. 2.4.

$$2.4 \qquad \sum_{i=1}^{n} \varepsilon i^2 = \sum_{i=1}^{n} \frac{(Ax_i - y_i + C)^2}{A^2 + 1} \triangleq F(A,C).$$

To calculate A and C in eq. 2.4, which determines the fit, the partial derivative of F(A,C) with respect to A and then C is set to 0. This will yield 2 equations of 2 unknowns, A and C. If the determinant of the resulting matrix does not vanish (i.e. two different equations are produced) we can solve for A and C to get the optimal fit. It can be easily shown, and will be, that the determinant never vanishes. [1]

3. Formulation Of The Fit Coefficients

The calculation of the partial derivative of F(A,C), eq. 2.4, with respect to A and C will be shown now

$$3.1 \qquad \frac{\delta F(A,C)}{\delta C} = \delta\left(\sum_{i=1}^{n} \frac{(Ax_i - y_i + C)^2}{A^2 + 1}\right) = \frac{2}{A^2+1} \sum_{i=1}^{n} (Ax_i - y_i + C) = 0$$

3.2 $$\frac{\delta F(A,C)}{\delta A} = \frac{\left(\sum_{i=1}^{n} \frac{(Ax_i - y_i + C)^2}{A^2 + 1}\right)}{\delta A}$$

$$= \sum_{i=1}^{n} \frac{2\,(Ax_i - y_i + C)\,x_i\,(A^2 + 1) - (Ax_i - y_i + C)^2\,2A}{(A^2 + 1)^2}$$

$$= \frac{2}{A^2 + 1} \sum_{i=1}^{n} \left[(Ax_i - y_i + C)\,x_i - (Ax_i - y_i + C)^2\,A\,(A^2 + 1)^{-1}\right] = 0$$

Since the partial derivatives are set to 0 the term $\frac{2}{A^2 + 1}$ can be cancelled out of both equations. Rewriting 3.1

3.1a $$\frac{\delta F(A,C)}{\delta C} = A \sum_{i-1}^{n} x_i - \sum_{i=1}^{n} y_i + nC = 0;\ \text{where}$$

n is the number of data points. Similarly for eq. 3.2 and also multiplying $(A^2 + 1)$ we get

3.2b $$\frac{\delta F(A,C)}{\delta A} = \sum_{i=1}^{n} \left[(Ax_i^2 - y_i x_i + Cx_i)\,(A^2 + 1) - (Ax_i - y_i + C)^2\,A\right]$$

$$= \sum_{i=1}^{n} \left[Ax_i^2 - y_i x_i + Cx_i - Ay_i^2 - AC^2 + A^2\,x_i y_i + 2ACy_i - A^2 Cx_i\right] = 0$$

This can be written into a form similar to eq. 3.1a.

3.2b $$\frac{\delta F(A,C)}{\delta A} = A \sum_{i=1}^{n} x_i^2 - A \sum_{i=1}^{n} y_i^2 + A^2 \sum_{i=1}^{n} x_i y_i + 2AC \sum_{i=1}^{n} y_i$$

$$- A^2 C \sum_{i=1}^{n} x_i - AC^2 n + C \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i y_i = 0$$

Notation can be introduced to reduce some bulk of the equations.

Let

$$\sum_{i=1}^{n} x_i \triangleq X, \quad \sum_{i-1}^{n} y_i \triangleq \dot{Y}, \quad \sum_{i=1}^{n} x_i^{\,2} \triangleq \overline{X},$$

$$\sum_{i=1}^{n} y_i^{\,2} \triangleq \overline{Y}, \text{ and } \sum_{i=1}^{n} x_i y_i \triangleq \overline{\overline{XY}}.$$

Since all five notations above are summations on the given data they can be considered constants in the equations. The P.D.'s can now be displayed as

$$3.3 \qquad \frac{\delta F(A,C)}{\delta A} = A\dot{X} - \dot{Y} + nC = 0$$

$$3.4 \qquad \frac{\delta F(A,C)}{\delta A} = A\,(\overline{X} - \overline{Y} + A\,\overline{\overline{XY}}) + AC\,(2\dot{Y} - A\dot{X} - Cn) + C\dot{X} - \overline{\overline{XY}} = 0$$

From 3.3 we can define C as

$$3.5 \qquad C = \frac{\dot{Y} - A\dot{X}}{n},$$

and inserting this into 3.4 and expanding

$$3.6 \qquad \frac{\delta F(A,C)}{\delta A} = A\overline{X} - A\overline{Y} + A^2\,\overline{\overline{XY}} + \frac{A\dot{Y}^2}{n} - \frac{A^2\dot{X}\dot{Y}}{n} + \frac{\dot{X}\dot{Y}}{n} - \frac{A\dot{X}^2}{n} - \overline{\overline{XY}} = 0$$

Combining into like terms w/r to A, a quadratic is formed.

$$3.6a \qquad \frac{\delta F(A,C)}{\delta A} = A^2 \left[\overline{\overline{XY}} - \frac{\ddot{XY}}{n} \right] + A \left[\bar{X} - \bar{Y} + \frac{\dot{Y}^2 - \dot{X}^2}{n} \right] + \left[\frac{\ddot{YX}}{n} - \overline{\overline{XY}} \right] = 0$$

The quadratic formula can be employed to solve for A. This will yield a minimum and a maximum value for A. A double root is impossible to be derived. This can be concluded because the square term is the negative of the constant in eg. 3.6a and because of the nature of the quadratic equation.

Placing the two unique values for A in eq. 3.5, two unique values for C are obtained. The two resulting equations are linearly independent because of unique A values and the nature of eq. 3.5. Thus, the determinant of the generated 2x2 matrix is non-zero and we can solve for an optimal fit.

Taking the second P.D. with respect to A, we derive the test for the value which minimizes the error.

$$3.7 \qquad \frac{\delta^2 F(A,C)}{\delta A^2} = 2A \left[\overline{\overline{XY}} - \frac{\ddot{XY}}{N} \right] + \left[\bar{X} - \bar{Y} + \frac{\dot{Y}^2 - \dot{X}^2}{N} \right]$$

$$3.8 \qquad \frac{\delta^2 F(A,C)}{\delta C^2} = N > 0$$

By taking the minimum produced by eq. 3.7 and placing it in equation 3.5, we obtain C, knowing C will be a minimum by eq. 3.8. The fit coefficients are derived and are of the form

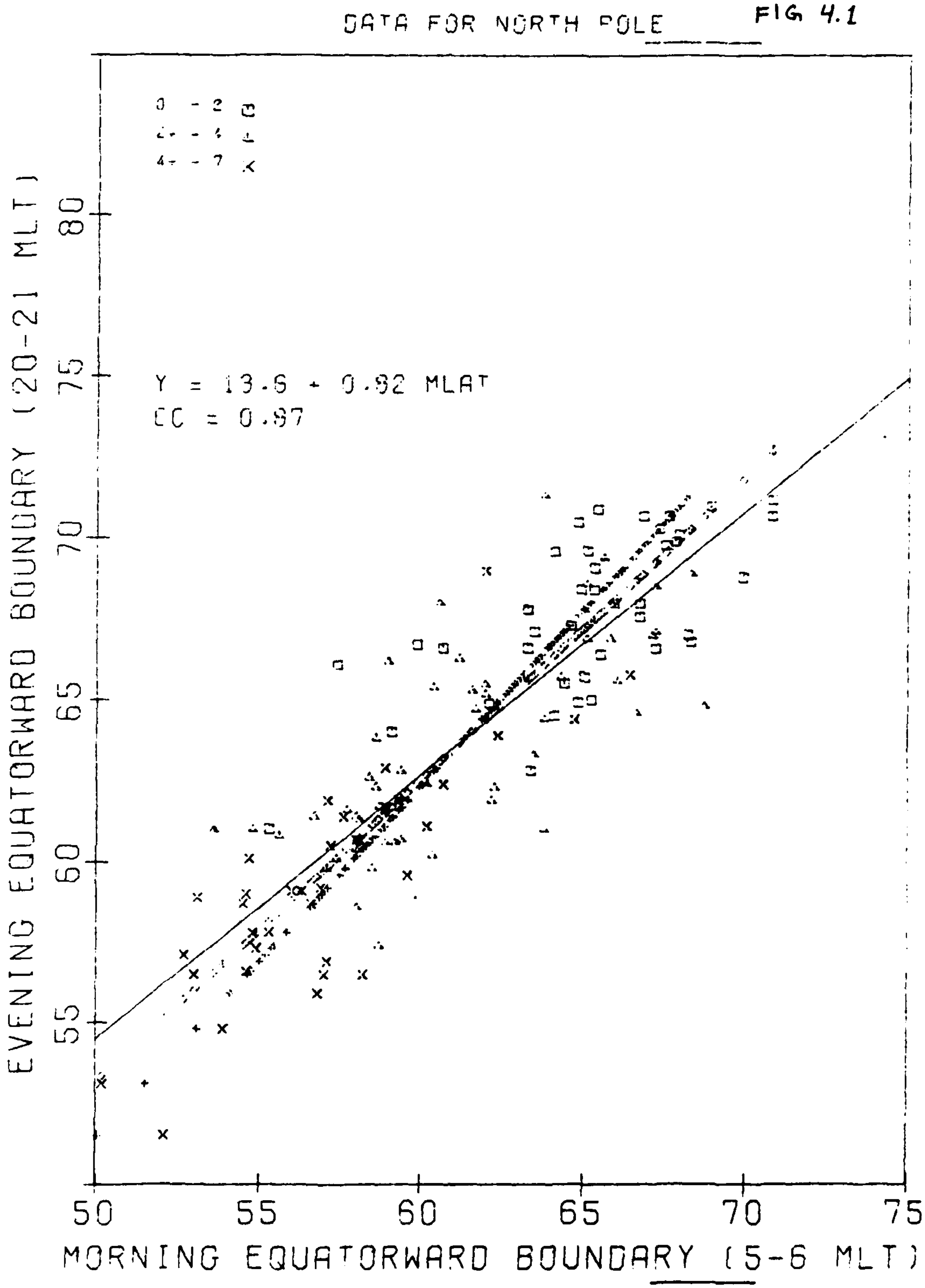
$$3.9 \qquad Y = Ax + C.$$

4. Applied Linear Fits

In Figure 4.1, data is plotted with the morning boundary as the abscissa and the evening boundary as the ordinate. Figure 4.2 is the same data only reversed coordinate system. The straight line is the normal least squares fit with x the independent variable. The line is determined by the "z's" is x and y independent and the line of "x's" is y independent. In each plot the line determined by x and y being independent falls between the other two lines as should be expected.

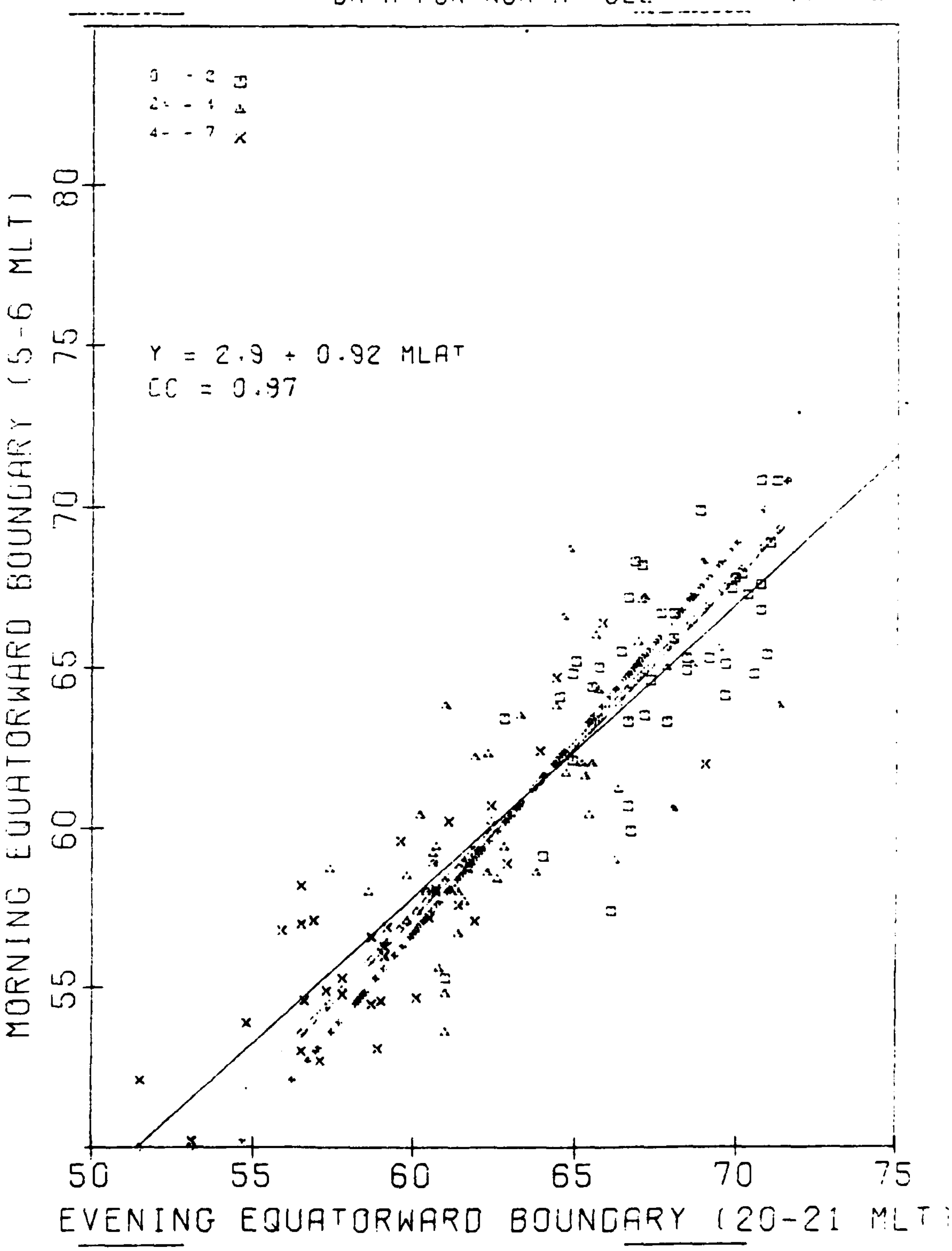
Figures 4.3 and 4.4 show similar results except for different magnetic local times and hemisphere.

FIG 4.1



DATA FOR NORTH POLE
0 - 2
2+ - 4
4+ - 7
Y = 13.6 + 0.82 MLAT
CC = 0.87
EVENING EQUATORWARD BOUNDARY (20-21 MLT)
55
60
65
70
75
80
50
55
60
65
70
75
MORNING EQUATORWARD BOUNDARY (5-6 MLT)

FIG. 4.2


DATA FOR NORTH POLE
0 - 2
2+ - 4
4- - 7
Y = 2.9 + 0.92 MLAT
CC = 0.97
MORNING EQUATORWARD BOUNDARY (5-6 MLT)
EVENING EQUATORWARD BOUNDARY (20-21 MLT)
50
55
60
65
70
75
80

DATA FOR SOUTH POLE FIG 4.3

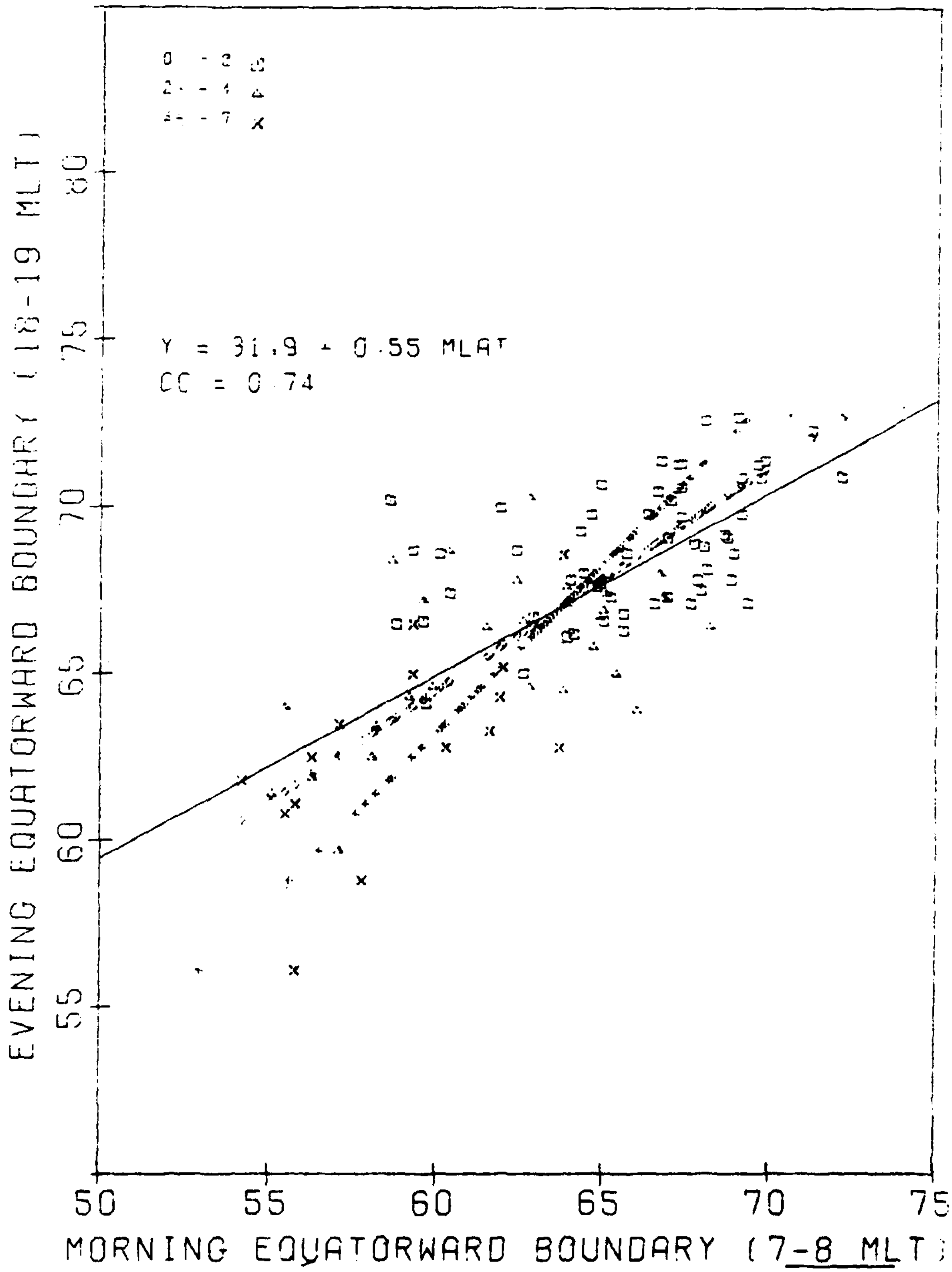
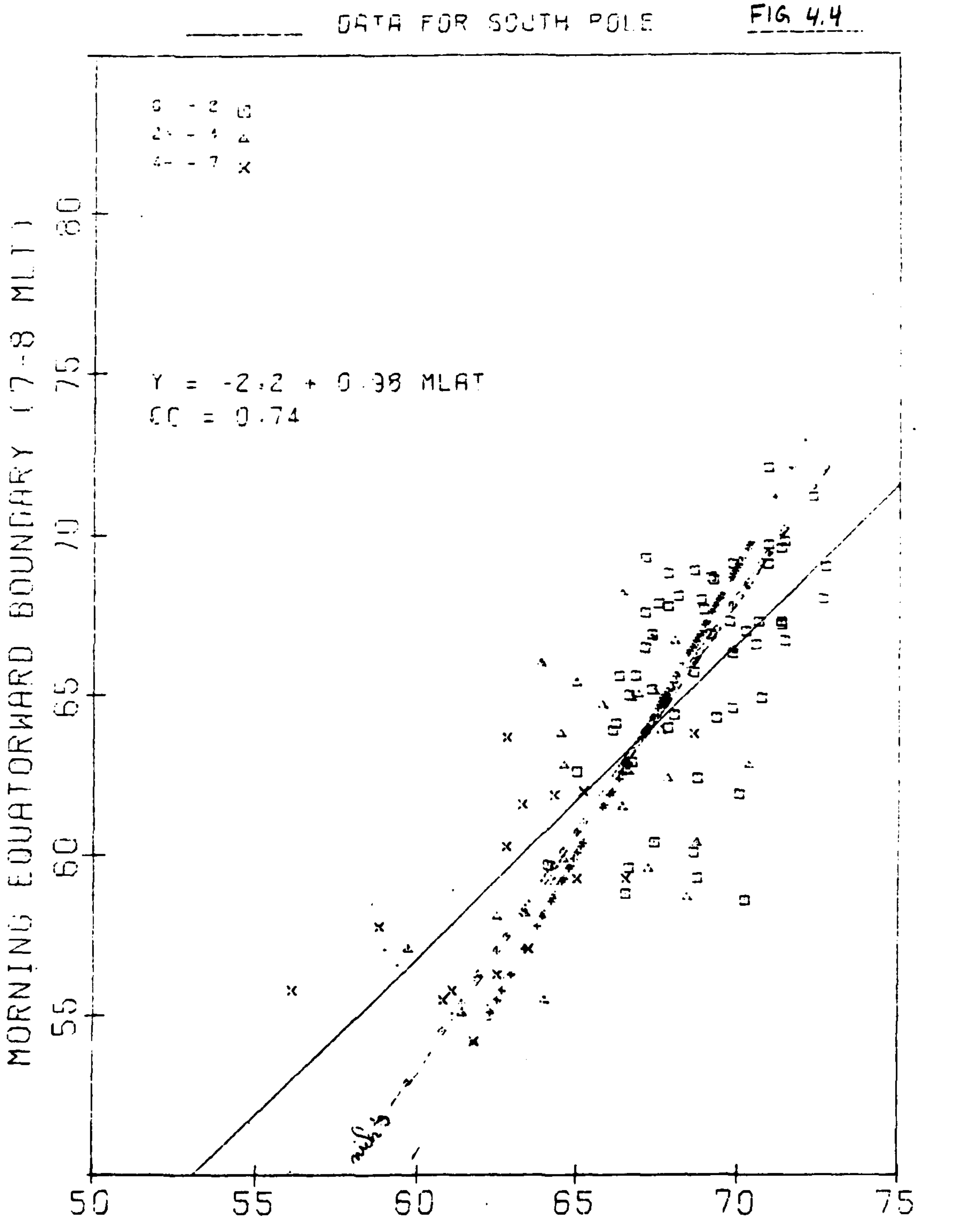


FIG 4.4



DATA FOR SOUTH POLE
0 - 2 □
2+ - 4 △
4- - 7 ×
Y = -2.2 + 0.98 MLAT
CC = 0.74
MORNING EQUATORWARD BOUNDARY (7-8 MLT)
EVENING EQUATORWARD BOUNDARY (18-19 MLT)
55
60
65
70
75
80
50

REFERENCES

1. Ralston, A. and Rabinowitz, P. (1965) - Numerical Analysis, McGraw-Hill, p. 250.